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Interdependent processes in high temperature deformation

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Relationships between stress and strain-rate are examined where two processes significantly influence high temperature deformation. The case when the two processes are interdependent is examined in detail and a gradual transition from one process to the other is predicted. Processes that show an interdependent transition are illustrated by data for the creep of polycrystalline Fe_2O_3 and for superplastic deformation.

1. Introduction

The relationship between the creep rate ($\dot{\epsilon}$) and applied stress (σ) for polycrystalline Fe_2O_3 shows an unusual feature (Fig. 1) in that the slope n decreases with increasing strain-rate [1].* This is not simply a result of testing technique or of material preparation because it has been seen in four-point bend creep [1], tensile creep [2] and compressive stress relaxation [3] and in both hot pressed and in cold pressed and sintered materials. Results such as those shown in Fig. 1 can appear to give non-integral exponents close to unity; these have been measured in creep of Fe_2O_3 [1] and in many other oxides, e.g. in Al_2O_3 [4], MgO [5], and ZrO_2 [6].

Experiments over a wide range of stress show that the curve for Fe_2O_3 is, in fact, "S" shaped [1, 3] and that the highest slopes are found at high stresses. This high stress region has been described by many authors and attributed to the control of the deformation rate by dislocation motion [7] or by intergranular cracking [6]; it will not be considered in detail here. Similar "S" shaped cruves have been found in superplastic materials [8].

An interpretation of the behaviour shown in Fig. 1, based on a threshold stress for diffusion creep, has been attempted [1, 2, 9]. This explanation has proved unsatisfactory because the curvature persisted after a threshold stress has been removed. Significant creep rates were measured below the apparent threshold stress and different, but equally convincing, threshold stresses were obtained with the same data over different ranges of stress.

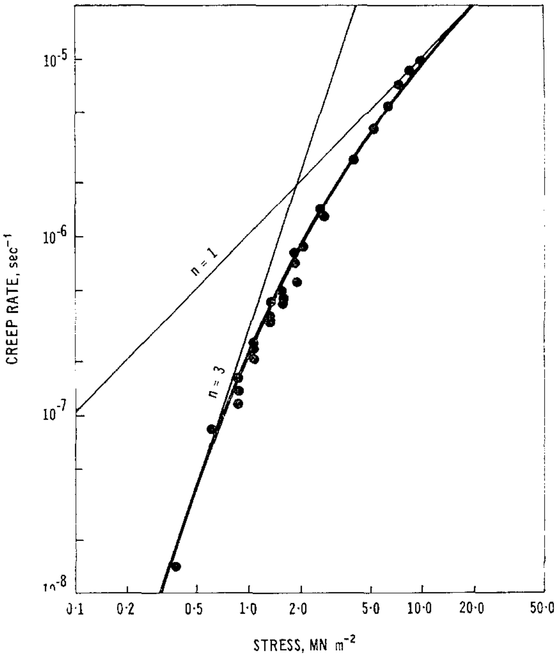

Figure 1 Data for the tensile creep of 84.3% dense Fe_2O_3 at 1100°C [2], fitted to Equation 7. Grain size 12 μm, $k_1 = 1.4 \times 10^{-6}$, $k_2 = 3.4 \times 10^{-7}$, $n = 3$.

No single deformation process is known that

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*We use the convention commonly adopted for oxide creep that the slope $n = \mathrm{d} \log \dot{\epsilon}/\mathrm{d} \log \sigma$; the reciprocal of this exponent is often used in describing superplastic materials and is denoted by m.

will give a curve such as Fig. 1. However, the shape of this curve can be described by a combination of two processes in each of which the creep rate depends on stress to some simple power, i.e. $\dot{\epsilon} = k\sigma^n$. Any tangent to Fig. 1 extrapolates to a strain-rate at a given stress that is faster than observed experimentally at that stress. This suggest that the slower process controls the overall rate and thus that the two processes are interdependent [10] (see Section 2.2).

A quantitative relationship between applied stress and resultant creep rate is derived here for such a combination of processes. This relationship is shown to be consistent with experimental data.

2. Relationships between stress and strain-rate

We define effective stresses σ_1 and σ_2 which drive the two processes and cause creep rates $\dot{\epsilon}_1$ and $\dot{\epsilon}_2$ respectively; this definition of strain-rate will be examined more closely in Section 2.2. To illustrate the way total stress and strain-rate should be related, we take the following relationships for the processes:

$$\text{process 1: } \dot{\epsilon}_1 = k_1\sigma_1 \quad (1)$$

$$\text{process 2: } \dot{\epsilon}_2 = k_2\sigma_2{}^n \quad (2)$$

where k_1 and k_2 are constants for a particular experimental condition but depend on grain size, temperature and other physical variables. Empirical power laws such as Equation 2 are accepted, over limited stress ranges, for high temperature deformation controlled by a single mechanism and often have a theoretical justification [7, 11-13]. Their applicability when several related processes occur is not established but to illustrate the method we will assume them to be valid. Any specific mechanism developed as a result of this analysis should be consistent with this assumption.

Provided that no additional mechanism generates heat or otherwise dissipates energy the work done by the applied stress should equal the work done by the two processes. In steady state creep (at constant stress) the work done per unit volume is $U = \sigma\epsilon$ so it follows that:

$$\sigma\dot{\epsilon} = \sigma_1\dot{\epsilon}_1 + \sigma_2\dot{\epsilon}_2 \,. \quad (3)$$

Special cases of this relationship are considered below.

2.1. Independent processes

If the two processes are totally independent each experiences the full applied stress and $\sigma_1 = \sigma_2 = \sigma$; substitution of this relationship and Equations 1 and 2 into Equation 3 gives:

$$\dot{\epsilon} = k_1\sigma + k_2\sigma^n \,. \quad (4)$$

This relationship, for totally independent (parallel) processes, has been derived previously (e.g. reference [11] and will not be considered further. It describes the creep rate in the high stress region of the "S" shaped curve referred to above.

2.2. Interdependent processes

If the two processes are interdependent, so that the operation of one is a requirement for the continuation of the other [10], then

$$\dot{\epsilon}_1 = \dot{\epsilon}_2 = \dot{\epsilon} \,. \quad (5)$$

This relation is necessary to ensure geometrical integrity of the specimen. It can be shown that any detailed geometrical factors, depending on the model for a process, can be contained within the constants k_1 and k_2. This is considered in more detail in the Discussion.

Equations 3 and 5 are satisfied if

$$\sigma = \sigma_1 + \sigma_2 \quad (6)$$

i.e. the sum of the effective stresses across each obstacle is the applied stress. Substitution of Equations 1 and 2 into Equation 6 gives:

$$\sigma = \dot{\epsilon}/k_1 + (\dot{\epsilon}/k_2)^{1/n} \,. \quad (7)$$

This relationship is shown schematically in Fig. 2. The creep rates are slower than predicted for the individual processes because the applied stress has to do work to overcome the other obstacle. It follows that examination of any limited stress range will indicate stress exponents that lie between the limits of n and unity. The rate of change of gradient of Equation 7 decreases as the stress increases; as a result a limited range of data at high stress may appear linear and extrapolate to an apparent threshold stress see Fig. 2.

This treatment can be readily extended to predict creep rates when three or more processes operate by including additional terms in Equation 3; this includes combinations of interdependent and independent processes. The use of Equations 1 and 2 to describe the processes does not restrict the method, and models which give other forms of these equations can also be used.

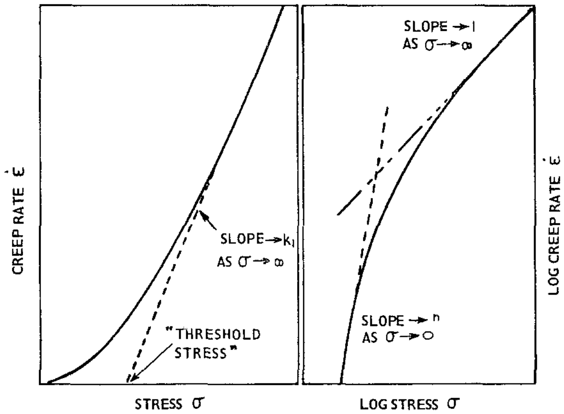

Figure 2 Relationship between creep rate and stress for interdependent processes, on linear and logarithmic axes.

3. Discussion

The solid line shown in Fig. 1 is a fitted curve of the form of Equation 7 using the best values of k_1, k_2 and n for Fe_2O_3. The agreement with experimental data points is good. There is evidence that effects similar to those shown in Fig. 1 can occur in UO_2 [14].

Similarities between oxide creep and superplastic deformation have been noted in the Introduction. Equation 7 has also been tested against literature data on superplasticity, considering only the low stress regions. An example is shown in Fig. 3 for a Pb 38.1 wt % Sn alloy deformed at room temperature [15]; the agreement is again good. Substituting $k_1 (= \dot{\epsilon}/\sigma)$ into the Coble [16] equation gives a product of boundary diffusivity and width $D_b w \sim 4 \times 10^{-21}$ m^3 sec^{-1}. This valve compares favourably with diffusivity-width products measured by tracer methods of 1.6×10^{-21} m^3 sec^{-1} for lead and 3×10^{-22} m^3 sec^{-1} for tin [17]. If Nabarro-Herring creep [18, 19] is assumed, there is a discrepancy of several orders of magnitude between the diffusivities. This agrees with the conclusion of Zehr and Backofen [17] that the deformation of Pb–Sn is controlled by Coble creep in the region of minimum stress dependence.

We have examined insufficient superplastic data to determine whether or not the consistency with Equation 7 can be considered general behaviour. Gifkins [20] has considered correctly the combination of independent processes in superplastic deformation ([11] and Equation 4), but his approximation for combining sequential or interdependent processes is inadequate. He assumes that, for $\dot{\epsilon} < \dot{\epsilon}_0$ (Fig. 4), $\dot{\epsilon}_2$ rate controls; process 1 can then be neglected so that

$$\dot{\epsilon} = k_2 \sigma^n , \quad \dot{\epsilon} < \dot{\epsilon}_0$$

similarly at higher creep rates

$$\dot{\epsilon} = k_1 \sigma \quad \dot{\epsilon} > \dot{\epsilon}_0 .$$

Using this combination, Gifkins anticipates a discontinuity of slope in the logarithmic plot, which is indicated in Fig. 4 with log and linear plots. This model for the combination of interdependent processes is valid at stresses and strain-rates sufficiently removed from $\dot{\epsilon}_0$ but is

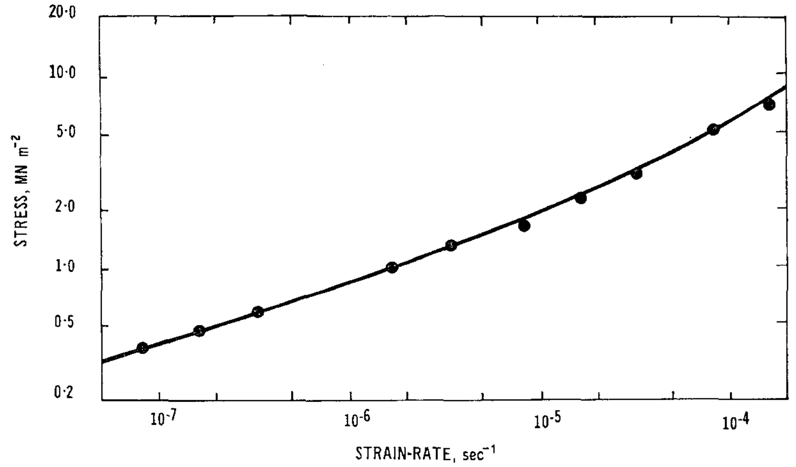

Figure 3 Equation 7 fitted to data for the superplastic deformation of Pb 38.1 wt% Sn at room temperature [15]. Grain size 2 µm, $k_1 = 5.4 \times 10^{-5}$, $k_2 = 1.7 \times 10^{-6}$, $n = 3$. The axes have been reversed with respect to Fig. 1 in accordance with the convention used in superplasticity.

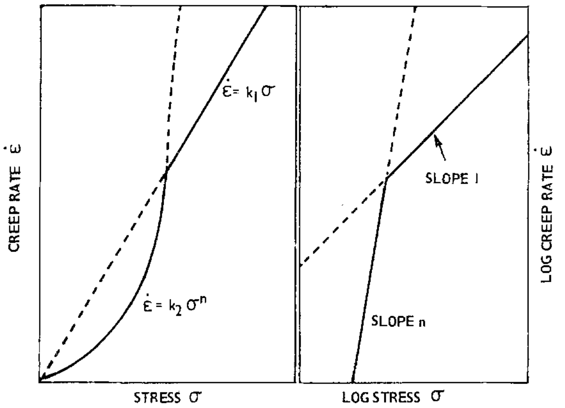

Figure 4 Relationship between creep rate and stress predicted by Gifkins [11] for interdependent processes, on linear and logarithmic axes.

incorrect when the creep rates for the two processes are comparable [10]. In the example that Gifkins considers [20], a more general form of Equation 7 should have been used to take account of the possibility of a number of competing independent processes in the higher stress region.

Besides Gifkins [20], other phenomenological theories have been developed to explain the strain-rate characteristics of superplastic alloys. Zehr and Backofen [17] obtained a relationship similar to Equation 7 from a dashpot model that does not adequately describe the flow of matter, they did not examine the justification of the analogue. Mechanical (and electrical) analogues can be constructed to give our Equation 7. Hart [21] used a mechanical analogue model which, although mathematically consistent, was difficult to reconcile with any real physical process and requires an additional mechanism to satisfy experimental observations [22]. Seeger specially excludes consideration of the transition region in his discussion [10] but notes that transitions will be over a range of stress, and will not occur abruptly.

Ashby and Verrall [23] have developed a model of diffusion accommodated flow to explain superplasticity. In its simplest form the model leads to a threshold stress below which no creep occurs. An apparent threshold stress can be defined for Fe_2O_3 but this is considered an approximation to Equation 7 rather than a true limiting condition. Ashby and Verrall developed other relationships which are similar in form to our Equation 7 and provide an example of a detailed model that fulfills the conditions laid down in Section 2.

In general, care should be taken when relating particular models with equation 5. For example, if Equation 1 is used to describe Nabarro-Herring [18, 19] or Coble [16] creep and Equation 2 describes the necessary grain-boundary sliding, it has been shown [24, 25] that approaching half of the total specimen deformation is a consequence of diffusion creep and the rest is a result of sliding. In this case Equation 5 could be written in the form $\dot{\epsilon} = \alpha_1\dot{\epsilon}_1 = \alpha_2\dot{\epsilon}_2$ and to obtain $\dot{\epsilon} = \dot{\epsilon}_1 + \dot{\epsilon}_2$, the constraint that $1/\alpha_1 + 1/\alpha_2 = 1$ must be included. However, it is simpler if α_1 and α_2 are included in k_1 and k_2 and thus it should be noted that such factors must be included in Equations 1 and 2 when considering interdependent flow.

In practice the technique described in Section 2 has two further advantages. For the processes considered here, only three simple parameters are involved and these are directly related to creep processes through Equations 1 and 2. These parameters can be evaluated over a limited range of stress by a curve fitting technique. In other models [20, 21] the parameters are determined from asymptotes to the experimental curves; this techniques requires data over a large range of stresses.

4. Conclusion

Relationships between creep rate and stress have been examined where two processes control creep. Independent and interdependent cases are characterized. The creep of Fe_2O_3 at low stress is consistent with a model where interdependent processes operate. The same model adequately describes the superplastic deformation of some alloys.

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